DOCUMENT RESUME

ED 279 687 TM 870 074

AUTHOR Marshall, Sandra P.
TITLE Understanding Arithmetic Story Problems.
PUB DATE 86
NOTE 19p.; One of 46 papers commissioned by the Study Group on the National Assessment of Student Achievement and cited in Appendix B to their final report "The Nation's Report Card" (TM 870 049). For other papers in this group, see TM 870 050-094.
PUB TYPE Viewpoints (120)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Arithmetic; *Cognitive Measurement; Educational Assessment; Educational Testing; Elementary Education; *Mathematics Tests; National Surveys; *Problem Solving; Scoring; *Test Format; Test Items; *Word Problems (Mathematics)
IDENTIFIERS *Free Response Test Items; National Assessment of Educational Progress

ABSTRACT

Open-ended individually administered questions can ascertain whether students can reason about arithmetic problems. Free response test items are useful in assessing students' thought processes as they solve mathematics story problems. Since story problems do not state explicitly which arithmetic operations are required for solution, students must decide to use a particular operation. Higher order cognitive skills are used in solving such problems. Different strategies may be used to solve a problem, reflecting different understandings of the problem. Appropriate schema--a memory structure describing common elements in seemingly different situations--are necessary to solve problems. To assess use of appropriate schema knowledge, it is necessary to know which information is considered by the student to be important in solving the problem. Test items can ask students to state what is important directly; to act out the problem using objects or drawings; to group problems according to similarity; to retell the problem; or to construct similar items. The emphasis in the paper is on story problems typical of first through eighth grade arithmetic. Six story problems, especially appropriate for grades 3 and 6, are included to illustrate the measurement of understanding of strategies and problem solving knowledge. (GDC)

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ED279687

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Paper commissioned by

THE STUDY GROUP ON THE NATIONAL ASSESSMENT OF STUDENT ACHIEVEMENT

1986

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Understanding Arithmetic Story Problems

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One purpose of open-ended, individually-administered questions is to assess whether students can reason about problems. Rather than having students compute answers to numerical questions, we now want to ask whether students understand why they carry out particular arithmetic operations and to probe their reasoning for so doing. While this information can be assessed to a limited extent by traditional problems in multiple-choice format, it is better addressed by certain kinds of free-response items.

A major advantage of open-ended questions is that we can determine not only whether the individual responds correctly but also what he or she misunderstands. Thus, there is diagnostic information present in these items as well as assessment information. A strong argument can be made for the value of the diagnostic information in a national assessment. For several years there have been reports that students have difficulty with higher-order cognitive skills, but no one has yet reported the nature of these difficulties. If we expect remediation in the classrooms, we must provide teachers, educators, and administrators with information about what students do and do not know, including details of student misconceptions and misunderstandings.

Diagnosis is particularly important when assessing higher-order cognitive skills, because the focus is not upon the content area (or facts) but upon processing, analyzing, and synthesizing available information. Evaluating the presence or absence of particular knowledge is less helpful here than assessing whether individuals have learned how to make inferences, follow logical arguments, reason analogically, interpret data, or summarize findings.

In this paper, I discuss the value of open-ended questions in assessing student reasoning about arithmetic story problems. I base my comments upon my own and others' recent research (Marshall, 1986a, 1986b, 1984, 1982; Mayer, 1982; Riley, Greeno, & Heller, 1983; Hinsley, Hayes, & Simon, 1977). There are three sections to the paper. The first contains a description of the domain. The second discusses theoretical aspects that should be considered. The third section presents examples of items with suggestions about how responses could be evaluated and scored.

The Domain

The domain of interest is arithmetic story problems. Story problems span much of the mathematics curriculum, beginning with elementary arithmetic problems in the first grade and evolving into algebra and calculus word problems of high school years. Story problems provide students the opportunity to apply computational skills within different settings. An important feature of story problems is that they do not state explicitly which arithmetic operations are required for solution. Students must make the decision to use a particular operation.

Typical examples of story problems at three levels of mathematics are:

Third Grade: There were 7 boys. Each boy had 3 cookies. How many cookies were there? [*California Assessment Program* item]

Sixth Grade: It is 1.3 kilometers from Alice's house to school. She rides her bicycle to and from school every day. How far does she ride in 5 days? [*California Assessment Program* item]

Algebra: One electronic reader can read a deck of computer cards in one-half the time required by another reader. Together, they can read the deck in eight minutes. How long would it take each reader alone to read the deck of cards? [*Algebra 1*, by Dolciani]

The emphasis in this paper is on story problems that are typical of first through eighth grade arithmetic. Algebra word problems will not be discussed in detail.

Higher-order cognitive skills involved in these problems include making inferences, using related and already known situational information, finding and recognizing

analogous items stored in memory, and following logical arguments. At the core of higher-order skills is the ability to understand at a general level what is going on in the item. Included in this level would be recognition of what question is asked, which variables ought to be considered and which are irrelevant, and whether the question is reasonable and logical. A person having this level of understanding should be able to restate the problem in general terms, describing the situation and the probable path to solution.

The Theory

There seems to be consensus among researchers that there are two general parts to solving a story problem (e.g., Marshall, 1986b; Mayer, 1981). The first is the representational stage, and the second is the computational stage. Higher-order skills reside in the first of these: understanding the problem and determining what to do in order to solve it. In the computational stage, procedural skills are invoked as the individual follows arithmetic algorithms of addition, subtraction, multiplication, or division.

What does it mean to understand a problem? There are several levels of understanding that can be assessed. The first focus of assessment is on the individual's choice of problem-solving strategy. Several distinct strategies have been observed in the problem solving of elementary school children. I have described three of them in detail elsewhere (Marshall, 1982). The strategies differ in their reliability, and they represent different orders of high-level cognition. A brief description of each one is given below.

Under one observed strategy, the individual may indeed be reading carefully and understanding the general structure of a given problem. The individual is clearly making an attempt to understand what's going on in the problem. In a second strategy, the individual may be using a "short-cut" strategy by attending only to selected key words (e.g., the words "have left" indicate subtraction, the word "altogether" means addition). In this strategy, the individual has become so preoccupied with the operation to be performed that the meaning of the variables has been lost. Finally, in a third approach,

the individual may be governed by a process defined as "visually moderated sequences" (Davis & McKnight, 1979). That is to say, the individual reads part of the problem, forms an equation and carries out the necessary operation, reads another part of the problem, operates upon it as required, and so on to the end of the problem.

Any of the three strategies may yield the correct answer but they reflect different understandings. In the first, the individual has a broad understanding of what's happening in the problem. In the second, the individual may be misled by reading only selected words. For example, in the strategy study cited above, I found that sixth-grade students were more likely to respond 11 2/3 to the following problem than they were to respond correctly:

> John has 12 baseball cards. He gave 1/3 of them to Jim. How many cards does John have left?

They were untroubled by the concept of 2/3 of a baseball card. In an extreme example of this same strategy, Schoenfeld (1982) describes a situation in which students elected to subtract, given a problem that began with the words "Mr. Left . . ."

In the third strategy, there is little attempt to understand the logical structure or to perceive the problem as a whole, and the individual fails to recognize inconsistencies in the problem. This strategy is not restricted to elementary school children. For example, consider the following:

> The number of quarters a man has is seven times the number of dimes he has. The value of the dimes exceeds the value of the quarters by $2.50. How many has he of each coin?

I have given this problem to students in my college cognitive psychology classes and asked them to find the solution. A large majority of the students engage in the third strategy: they attempt to write equations for the problem, apparently processing each statement independently. Only a small number realize immediately that the problem as stated is impossible.

It will be particularly important to identify use of the second and third strategies early in elementary school and to modify the ways in which students use them. Persistence in their use will cause difficulties as the students move into the more difficult

problems of algebra and calculus. At the least, it is important that students develop metacognitive skills of checking the reasonableness of a problem or a solution.

One goal of assessment is to identify the general nature of problem-solving strategies used by individuals. But this is just the beginning. Given the use of the superior top-level strategy—the attempt to understand a problem in the broad sense—there is also the question of how much the individual actually understands of the problem. This may be recast as a question of whether the individual is using an appropriate representation for the problem. This question brings in the issue of related world knowledge and experiences of the individual in situations similar to those portrayed in the story problems.

Current theories of knowledge acquisition, storage, and retrieval from memory posit the importance of situation-specific knowledge (e.g., Resnick, 1986). Research in cognitive psychology indicates that individuals remember events, situations, experiences and the important features associated with them (e.g., Anderson, 1985). As we present story problems to individuals, we are in part asking them to use their knowledge of the situation represented in the problem to fill in gaps that are not mentioned explicitly in the problem. For example, when we use algebra story problems about river currents and boating times up and down a river, we expect students to use stored knowledge about rivers, boats, methods of locomotion, and influence of river currents. The problems would have no meaning for someone who has no previous knowledge of these things.

Over time and with repeated exposure to various situations, we expect individuals to recognize that there are common features in many seemingly different situations and to build from these common features a new memory structure that sufficiently describes the common elements. This new structure is called a *schema*.

A schema consists of: (1) a collection of specific facts about the general situation, (2) a set of conditional statements that must be met if the schema truly represents the situation, and (3) a system of rules for using the schema structure itself. A well-known example of a schema is the restaurant schema. We recognize the situation by certain universal characteristics (e.g., tables, menus, waiters or waitresses, cashier), we

have certain expectations (e.g., the menus will contain lists of food items we can order, the waiter will not sit down to eat with us, we can select the food we want), and we follow a common set of rules (e.g., we use the information on the menu to order, we pay the bill, we tip the waiter). In any situation that evokes this particular schema, we do not use all the information about restaurants that we know. However, the situation must contain sufficient detail to trigger the restaurant schema rather than some other schema.

It can be argued that higher-order thinking corresponds in large measure to the use of higher-order knowledge structures in long-term memory. In which case, an assessment of these higher-level cognitive skills must address the issue of whether individuals have successfully formed high-level knowledge structures such as schema structures. There are several important psychometric questions involved in evaluating schema knowledge. I have discussed some of these in detail elsewhere (Marshall, 1986b). The issues include: how do we construct items that sample schema knowledge, what is the best item to elicit certain types of schema information, and how can we estimate the degree of connectivity of schema knowledge. These issues have not been fully resolved, but they are well enough understood to allow practical assessment of the development and use of higher-order cognitive skills.

To assess whether individuals are using appropriate schema knowledge, we must ask more probing questions that the usual one that asks for the correct numerical solution. It is valuable to know which information in the problem is considered important by the individual, to know whether the individual perceives similarities between the problem and others that have the same underlying schema structure, and to know whether the individual forms a plan for solving the problem.

These questions are not easily posed to students. Especially at the elementary grades, children are not usually able to verbalize the processes by which they solve problems. We need to design questions that elicit from them the desired information. This goal can be achieved in a variety of ways. Some possibilities are the following:

(1) We can inquire directly about the importance of information in any problem.

(2) We can ask students to make visible their understanding of a problem by acting out the problem with concrete, manipulable objects.

(3) We can have students diagram or draw the situation depicted in a problem.

(4) We can ask students to group problems according to similarity.

(5) We can require that students restate or tell us the problem using their own words.

(6) We can have students construct problems that are similar to given problems.

It is important to evaluate responses on more than a correct/incorrect scale. An incorrect response can tell us much more about an individual's understanding than a correct one. We can often determine that an individual has the correct knowledge representation of the problem but has failed to identify the appropriate information in the problem required for solution. We may find out that the individual has the correct schema but lacks the ability to carry out the required steps for solution. Or we may find out that there are gaps, inconsistencies, or errors in the schema being accessed by the student. Each of these findings has important and different instructional implications.

Most researchers who study the acquisition and use of schema knowledge structures emphasize the relationship between this knowledge and procedural knowledge. For example, in John Anderson's LISP Tutor and Geometry Tutor, it is evident that procedures necessary for implementing schema knowledge will be embedded in or directly linked to that schema (Anderson, Boyle, Corbett, & Lewis, 1986). Procedural knowledge is usually conceptualized as a set of conditional statements, or rules, having the form: If A, then B. Carrying out a procedure corresponds to using these rules one at a time until a specified goal is obtained. Some examples of procedural knowledge are adding two numbers, constructing a geometry proof, or writing a computer program.

As we evaluate knowledge schemata, we have the opportunity to assess related procedural understanding at the same time. Some of the following examples illustrate how we can evaluate both types of knowledge with a single item.

Examples

The following items illustrate how we can investigate students' understanding of strategies and problem-solving knowledge through free-response assessment.

EXAMPLE 1: Story Problem, Third Grade

> The pet store had 4 cages of puppies. There were 3 puppies in each cage. The store sold 2 puppies. How many puppies were left?

The students' understanding can be assessed by posing questions like these:

1. Here are several objects (e.g., blocks, buttons). How would you act out the problem?

   [*This question tests explicitly the student's schema or internal representation of the problem without taxing limitations of short-term memory processing capacity.*]

2. Mary solved the problem like this:

   $$4 + 3 = 7 \text{ and } 7 - 2 = 5.$$

   Would you solve it in the same way? What do each of the numbers mean?

   [*This question requires analysis of both the problem and a possible solution. It provides information about the individual's ability to recognize incorrect representations and to understand why the solution is incorrect.*]

3. Now look at these problems. Which one do you think is most like the puppy problem? How is each one similar or not similar to the puppy problem?

   a. Last week I had 2 packages of cookies. Each package had 4 cookies in it. If I ate 1 of the cookies, how many cookies were there?

   b. The black dog had 3 black puppies and 4 white puppies. How many puppies did she have altogether?

   c. Mary had 12 marbles and she gave 4 of them to her friend Judy. How many marbles did she have left?

   [*Here we get information about which features are influencing the student's perception of the problem. In alternative a, the underlying structure of the problem matches the original problem. In problem b, the problems have different*

structure but are both about the same content. In alternative c, the problem structure is different, the content is different, but the question is the same: how many are left?]

4. Can you make up a problem that you think is similar to the puppy problem? Make your problem about Sam and his toy trucks. Show me what your problem looks like by using these toys (have on the table three toy trucks and several small objects that can be placed in the trucks).

   [*This question also focuses on the ability to represent the structure of the problem using manipulatives. It evaluates the student's ability to abstract the general form from the original problem and to create a new instance of the structure with new and unrelated objects, i.e., Sam and toy trucks.*]

EXAMPLE 2: Story Problem, Third Grade

Suppose you have three friends with you at your house. You want to share a bag of cookies with your friends. There are 15 cookies in the bag. How many cookies will you and each of your friends get?

1. Can you draw a picture to show what is happening in this problem?

   [*This gets at similar knowledge structures as using blocks, etc. to act out problem. A student's drawing reflects how he or she organizes the information in the problem.*]

2. Can everyone have the same number of whole cookies?

   [*This question examines the child's understanding of equal portions and of the part/whole relation.*]

3. How could everyone have the same number of cookies?

   [*This is a followup to question 2—it investigates whether the child realizes that cookies are objects that can be broken into smaller units.*]

4. What if the problem were about marbles. Could everyone have the same number of marbles? Why?

[Again, a followup to question 2. This question forces the child to examine the differences between things that can be partitioned and things that cannot. This taps the child's understanding of when we can meaningfully talk about "one half of something" and when we cannot.]

EXAMPLE 3: Story Problem, Sixth Grade

Alice had pet hamsters and canaries. One day, she counted the heads and feet of her pets. She counted 7 heads and 24 feet. How many hampsters and how many canaries did she have?

1. Joe worked this problem in the following way:

   $7\sqrt{24} = 3$

   Would you work it in the same way?

   [Framing the question in this way allows the child to understand that more than one approach might be taken.]

   a. If so, what does the 3 mean?

      [If the child agrees with the solution, it is important to know whether he or she understands or can interpret the results of the computation.]

   b. If not, can you explain to him why it is wrong?

      [This question allows us to follow the child's reasoning about the problem. This is not just allowing the child to explain how to solve the problem. Rather, the emphasis is on having the child identify the error and explain why it is an error.]

2. What extra information do you need to know in order to solve this problem? (that canaries have 2 feet each, hampsters have 4 feet each, and each pet has 1 head.)

   [Is the child aware that extra knowledge is needed here? Does the child have the knowledge about birds and mammals needed for this item?]

3. When Gerald solved this problem, he said that there were 4 canaries and 3 hamsters? Do you agree? Can you act this out with the objects on the table? (Have beans, buttons, etc. on the table)

   [*If the child has been unable to solve the problem, this question provides a clue about how he or she might go about solving it. It is important to see how the child deals with two variables (head and number of feet) per pet.*]

4. Can you describe a general plan for solving problems like this? What are some ways you could represent the information in the problem?

   [*For example, does the student mention drawings, diagrams, objects on the table? Does the student have the strategy of diagramming all combinations of 2 numbers whose sum is 7? Is the student using a trial-and-error or a systematic approach?*]

EXAMPLE 4: Story Problem, Sixth Grade[1]

The 130 students from Roosevelt School are going on a picnic. Each school bus holds 50 passengers. How many buses will they need?

Ellen worked the problem in the following way:

$$\begin{array}{r} 2 \\ 50\overline{)130} \\ \underline{100} \\ 30 \end{array}$$

1. What does the number 2 represent in the computation? How do you interpret it?

   [*The student should be able to explain that the 2 represents the number of times 50 is contained in 130. The student also needs to recognize that the 2 represents the number of buses that will be completely filled.*]

2. What does the number 30 mean? What does it mean in the solution to the problem?

[1] This problem is adapted from a multiple-choice item developed by the California Assessment Program, Annual Report, Sacramento, California, 1983.

[*This is a followup question. Again there are two points to be made. Technically, 30 is the remainder in the division problem. It also represents the number of students left after the first two buses have been completely filled.*]

3. Which is the better answer to the problem: 2 3/5 or 3? Why?

[*This questions gives up information about the student's understanding of remainder coupled with his or her understanding of part/whole. Since it is impossible to have 30/50 = 3/5 of a bus to accommodate the remaining students, it will be necessary to have another whole bus. The student needs to explain that although 2 3/5 is computationally correct, it is not a meaningful solution to the problem.*]

4. Draw your solution.

(after the drawing is complete)

a. How is the number 2 shown in your drawing?

b. How is the number 30 shown?

[*This question allows us to compare the student's verbal ability to describe the quantities in the problem with his or her ability to represent diagrammatically the same information. It is important to ascertain whether students may have full understanding of the problem without being able to describe it verbally.*]

EXAMPLE 5: Story Problem, Sixth Grade

STEALING PEACHES[2]

A naughty Boy named Augustus, aged 10 years and 7 months, entered a Farmer's seven-year-old peach orchard near Swallowville, and comsumed 3/4 of a bushel of freestone Peaches before being discovered by the Farmer, aged 42. The race was a mad one. Augustus barely escaped, carrying with him 2 peaches intended for his Grandma. When he reached a position of safety, he sat down to get his breath. He then found that in his flight he had lost from his

[2] This example is taken from R. Weeks, *Boy's Own Arithmetic*, 1924.

pockets 16 marbles which had cost him on an average 14 cents each. He had also torn his trousers, but he bore it bravely. In his depression over the loss of the marbles, he ate 1 of the 2 peaches. The other he presented later to his Grandma, who bought him 20 marbles at 17 cents each and mended his trousers. The Farmer picked up 5 of the lost margles which he sold at 11 cents each the next time he went to town. The purchaser was none other than Augustus himself, who was not recognized because he wore another pair of trousers and had his face washed. What was the net gain or loss to all three parties concerned? Estimate the peaches at 84 cents a bushel and leave the trousers out of account.

[*This problem is much more complex than typical problems solved by sixth-grade students. It has a complicated structure, and the student must pay attention to the story as well as to the numbers in the story. The examiner should record how the student organizes the material and what plan he or she makes. If possible, it would be useful to keep a record of the student's verbal protocol.*]

EXAMPLE 6: Algebra Story Problem

BRIGANDS IN SNUGGLETOP WOODS[3]

Dr. Thomas Turvy, aged 42, a country Physician living at Juniper Flats, had been called one July dat to visit a sick child in the hills. At 9:25 p.m. he telephoned his wife from a hamlet which went by the name of Jacob's Cats: "Am summoned by phone to Snuggletop. A lumberman has a broken leg and arm and perhaps internal injuries. Have fed my horse. Starting at once. The roads up and down these hills are terrible! Good night!"

Just 20 minutes later, a strange voice, that of a woman, spoke to Mrs. Turvey over the phone: "I don't dare tell you my name, but there is a plot to rob or to kill Dr. Turvey in Snuggletop Woods, and I overheard a man telephoning him a few minutes ago. You know where the Doctor is. Try to warn him!"

[3] This example is taken from R. Weeks, *Boy's Own Arithmetic*, 1924.

Mrs. Turvey hurried out into the night to find a messenger. She told the news to everyone she met. Charlie Chalmers, aged 12 years, heard her. Without saying anything, he ran to the river, jumped into his Father's canoe, and started rowing downstream just 5 minutes after the telephone call to Mrs. Turvey. He knew that ten miles down the river there was a path a quarter of a mile long which led to the Crossroads, where the Doctor would have to pass. He rowed at the rate of 3 1/2 miles an hour, and the current of the stream moved at the rate of 2 1/2 miles an hour. On reaching the path and tying up his boad, he found himself so weary that it took him 5 minutes to reach the Crossroads.

Charlie was not the only child at Juniper Flats who heard of the robbers' plot against good Dr. Turvey. Little Sadie Lee heard it. The night was dark, but she had a bicycle and rode well. What an adventure for a Girl only 11! She slipped out unseen, and started down the River Road 9 minutes after Charlie Chalmers had started on the river. She knew the path which led from the river to the Crossroads. The path crossed the River Road 9 miles from where Sadie started. Her speed was 7 1/2 miles an hour. On reaching the path, she pushed her bicycle into a bush, and commenced to stagger up the steep path. She was small and fat, and it took her 10 minutes to reach the Crossroads.

Obadiah Bunkover, aged 51, the well-known Tax Collector, who lived at Juniper Flats, was not the first to hear the news, but when he heard it, he decided to drive by horse and buggy over the Mountain Road to the Crossroads, a distance of 7 miles. He had a powerful horse named Mt. Shasta because he was so tall, who jerked the buggy along at a rate of 10 miles an hour. Unfortunately, a wheel came off 2 miles from Juniper Flats. Mr. Bunkover tried in vain for 11 minutes to put the wheel back, then junped on Mt. Shasta and rode bareback at the rate fo 12 miles an hour.

Dr. Turvey had started from Jacob's Cats at 9:25. His good old horse, named Paregoric, aged 12 years, made only 6 2/3 miles an hour over the mountainous roads. The distance to the Crossroads was 13 2/3 miles.

To reach the village of Snuggletop, one turns to the right at the Crossroads and follows for 3 miles a wild road which winds through Snuggletop Woods. Exactly in the center of this dark forest, 3 Brigands, heavily armed, were waiting . . .

The Doctor turned at the Crossroads and continued at the same speed. If he overtook one of the messengers after 1/3 of a mile on the terrible road to Snuggletop, who was this messenger, and what was this messenger's rate from the Crossroads to the point of meeting? If Mr. Bunkover arrived at the Crossroads at the exact moment when the Doctor overtook the messenger, when did Mr. Bunkover leave Juniper Flats? Where was the other messenger at the moment when Dr. Turvey received warning of the danger ahead of him in Snuggletop Woods?

[*This problem requires more thought than most algebra problems. It is included here to show that we can and should ask students to solve complex problems that require organization and careful attention to detail. The examiner should record whether the student constructs diagrams to aid in the solution, whether he or she develops tables or charts, whether there is a well-defined plan of solution, and whether the student accurately recognizes all of the variables and uses them correctly. The examiner will also have the opportunity to observe whether the individual has metacognitive skills that monitor the individual's progress and that are used to determine whether the individual is reaching a sensible solution. Verbal, talk-aloud solutions should be encouraged.*]

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